例10 求 $\lim\limits_{x\to-\infty}\left(\sqrt{4x^2-8x+5}+2x+1\right)$.

解 令$u=-x$,则 负代换

$$\text{原式}=\lim_{u\to+\infty}\underbrace{\left(\sqrt{4u^2+8u+5}-2u+1\right)}_{\infty-\infty}$$

$$=\lim_{u\to+\infty}\frac{12u+4}{\sqrt{4u^2+8u+5}+(2u-1)}$$

$$=\lim_{u\to+\infty}\frac{12+\frac{4}{u}}{\sqrt{4+\frac{8}{u}+\frac{5}{u^2}}+\left(2-\frac{1}{u}\right)}$$

$$=3$$

令$t=\frac{1}{u}$,则有

$$=\lim_{t\to0^+}\left(\sqrt{\frac{4}{t^2}+\frac{8}{t}+5}-\frac{2}{t}+1\right)$$

$$=\lim_{t\to0^+}\frac{\sqrt{4+8t+5t^2}+t-2}{t}$$

$$=\lim_{t\to0^+}\frac{12t+4t^2}{t[\sqrt{4+8t+5t^2}-(t-2)]}$$

$$=\lim_{t\to0^+}\frac{12+4t}{\sqrt{4+8t+5t^2}-(t-2)}$$

$$=\frac{12}{4}=3$$